Algebra $\frac{1}{2}$

An Incremental Development

THIRD EDITION

SAXON

Contents

LESSON 1 Whole Number Place Value • Expanded Notation • Reading and Writing Whole Numbers • Addition

1.A whole number place value

We use the **Hindu-Arabic system** to write our numbers. This system is a base 10 system and thus has ten different symbols. The symbols are called **digits,** or **numerals,** and they are

0, 1, 2, 3, 4, 5, 6, 7, 8, 9

The numbers we say when we count are called **counting numbers,** or **natural numbers.** We may show the set of counting numbers this way:

{1, 2, 3, 4, 5, ...}

The three dots, called an *ellipsis*, mean that the list continues without end. The symbols, { }, are called braces and are sometimes used to designate a set. If we include zero with the set of counting numbers, then we form the set of **whole numbers.**

{0, 1, 2, 3, 4, ...}

When we write whole numbers, we can write the **decimal point** at the end of the number, or we can leave it off. Both of these

427. 427

represent the same number. In the right-hand number, the decimal point is assumed to be after the 7.

The value of a digit in a number depends on where the digit appears in the number. The first place to the left of the decimal point is the ones' place. We also call this place the **units' place,** which has a **place value** of 1. The next place to the left of the units' place is the **tens' place,** with a place value of 10, followed by the **hundreds' place,** with a place value of 100, and then the **thousands' place,** with a place value of 1,000. Each place to the left has one more zero.

Whole Number Place Values

hundred trillions	ten trillions	trillions	,	hundred billions	ten billions	billions	,	hundred millions	ten millions	millions	,	hundred thousands	ten thousands	thousands	,	hundreds	tens	units	.	decimal point
100,000,000,000,000	10,000,000,000,000	1,000,000,000,000		100,000,000,000	10,000,000,000	1,000,000,000		100,000,000	10,000,000	1,000,000		100,000	10,000	1,000		100	10	1		

To find the value of a digit in a number, multiply the digit times the place value. For example, the 5 in the left-hand number below

$$415{,}623 \qquad 701{,}586 \qquad 731{,}235$$

has a value of 5×1000, or 5000, because it is in the thousands' place. The value of the 5 in the center number is 5×100, or 500, because it is in the hundreds' place. The value of the 5 in the right-hand number is 5×1, or 5, because it is in the units' (ones') place.

example 1.1 In the number 46,235:

(a) What is the value of the digit 5?

(b) What is the value of the digit 2?

(c) What is the value of the digit 4?

solution First we write the decimal point at the end of the number.

$$46{,}235.$$

(a) The 5 is one place to the left of the decimal point. This is the units' place. This digit has a value of 5×1, or **5.**

(b) The 2 is three places to the left of the decimal point. This is the hundreds' place. This digit has a value of 2×100, or **200.**

(c) The 4 is five places to the left of the decimal point. This is the ten-thousands' place. This digit has a value of $4 \times 10{,}000$, or **40,000.**

1.B expanded notation

Writing a number in **expanded notation** is a good way to practice the idea of place value. When we write a number in expanded notation, we consider the value of every digit in the number individually. To write a number in expanded notation, we write each of the nonzero digits multiplied by the place value of the digit. We use parentheses to enclose each of these multiplications and put a plus sign between each set of parentheses.

To write 5020 in expanded notation, we write

$$(5 \times 1000) + (2 \times 10)$$

because this number contains five thousands and two tens.

example 1.2 Write the following number in standard notation: $(4 \times 10{,}000) + (6 \times 100) + (5 \times 1)$

solution Standard notation is our usual way of writing numbers. The number has four ten thousands, no thousands, six hundreds, no tens, and five ones. The number is **40,605.**

example 1.3 Write the number 6,305,126 in expanded notation.

solution

There are six millions, $(6 \times 1{,}000{,}000)$	three hundred thousands, $(3 \times 100{,}000)$		
five thousands, (5×1000)	one hundred, (1×100)	two tens, (2×10)	and six ones. (6×1)

If we add them all together, we get

$$\mathbf{(6 \times 1{,}000{,}000) + (3 \times 100{,}000) + (5 \times 1000) + (1 \times 100) + (2 \times 10) + (6 \times 1)}$$

1.C reading and writing whole numbers

We begin by noting that all numbers between 20 and 100 that do not end in zero are hyphenated words when we write them out.

23 is written twenty-three	64 is written sixty-four
35 is written thirty-five	79 is written seventy-nine
42 is written forty-two	86 is written eighty-six
51 is written fifty-one	98 is written ninety-eight

The hyphen is also used in whole numbers when the whole number is used as a modifier. The words

ten thousand

are not hyphenated. But when we use these words as a modifier, as when we say

ten-thousands' place,

the words are hyphenated. Other examples of this rule are

hundred-millions' digit
ten-billions' place
hundred-thousands' place

The word *and* is not used when we write out whole numbers.

501	is written	five hundred one
	not	five hundred and one
370	is written	three hundred seventy
	not	three hundred and seventy
422	is written	four hundred twenty-two
	not	four hundred and twenty-two

Do not think of this as a useless exercise! Knowing how to correctly and accurately write numbers is necessary when writing a check, for example. Before we read whole numbers, we place a comma after every third digit beginning at the decimal point and moving to the left.[†] The commas divide the numbers into groups of three digits.

Place Value

Trillions			Billions			Millions			Thousands			Units (Ones)			Decimal point
Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones	.

To read the number 4125678942, we begin on the right-hand end, write a decimal point, and separate the number into groups of three by writing commas.

4,125,678,942.

Then we read the number, beginning with the leftmost group. First we read the number in the group, and then we read the name of the group. Then we move to the right and repeat the procedure.

four billion, one hundred twenty-five million, six hundred seventy-eight thousand,
nine hundred forty-two

[†] It is our convention to usually write four-digit whole numbers without commas.

Test Forms

Algebra $\frac{1}{2}$

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Test 1 **SHOW YOUR WORK** **Name:** _____

1. Subtract: $\begin{array}{r} 737 \\ -\ 668 \\ \hline \end{array}$

2. Add: $\begin{array}{r} 81{,}104 \\ 20{,}229 \\ +\ 18{,}284 \\ \hline \end{array}$

Multiply:

3. 273×87

4. $26 \cdot 3 \cdot 64$

Divide:

5. $\frac{\$30.10}{7}$

6. $\frac{\$21.55}{5}$

7. $\frac{\$19.39}{7}$

Multiply:

8. 274×47

9. $81 \cdot 6 \cdot 15$

Find the missing number:

10. $B \cdot 16 = 192$

11. $W - 197 = 365$

12. $917 - E = 540$

13. $B + 219 = 370$

14. $\frac{90}{W} = 9$

15. A number has nine digits. All the digits are 1 except the millions' digit, which is 5, the ten-thousands' digit, which is 2, and the tens' digit, which is 7. Use digits to write the number.

16. Use words to write the number 10134519.

17. Write the number 6,203,649 in expanded notation.

18. Write the following number in standard notation:

$$(2 \times 10{,}000) + (3 \times 1000) + (5 \times 100) + (5 \times 10) + (3 \times 1)$$

19. Round 23,372,931 to the nearest hundred thousand.

20. Arrange the following numbers in order from least to greatest:

$$-361, -80, 139, 44, -134, 229$$

Test 11 **SHOW YOUR WORK** **Name:** _____

1. Twenty-six kilograms of tea cost 234 dollars. How much tea could you buy for 414 dollars?

2. Ward 1 reported 5 times as many votes as Ward 3 reported. Ward 8 reported 9 times as many votes as Ward 3 reported. If Ward 3 reported 5155 votes, how many votes did the three wards report in all?

3. The average of the first 4 numbers was 15. The average of the next 6 numbers was 25. What was the overall average of the 10 numbers?

4. Graph the following points: (a) (2, 4) (b) (–8, 3) (c) (–6, –7)

5. Find LCM (14, 26, 351).

Solve:

6. $z + \frac{1}{8} = \frac{1}{4}$

7. $\frac{5}{7}k = \frac{5}{9}$

8. $4y = 40$

9. Evaluate: $xy + x$ if $x = 3$ and $y = 5$

10. Simplify: $4 \cdot 4 + 15 - 6 \div 2 + 3(9 - 6)$

11. Subtract: $8\frac{2}{7} - 5\frac{1}{3}$

Simplify:

12. $3\frac{3}{4} \times 2\frac{2}{3} \times 2\frac{1}{7}$

13. $1\frac{1}{7} \div 1\frac{1}{5} \cdot 1\frac{3}{4} \div 2\frac{1}{3}$

14. $13 - \sqrt[3]{27} + 9 \cdot 3^4 - (5 + 1 - 3) \div 3$

15. (a) $\sqrt[3]{729}$ (b) $\sqrt{256}$

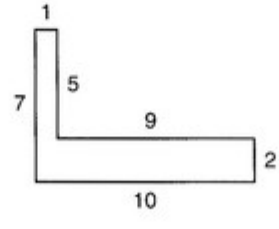
16. Multiply: 1.76×26.8

Simplify:

17. $3[(11 - 4)(15 - 11) + 7] - 9$

18. $38 - 7 \cdot 2 + 7 \cdot 3$

19. Find the perimeter of this figure. Dimensions are in inches. All angles are right angles.



20. Use a unit multiplier to convert 817 centimeters to meters.