

The cover features a complex geometric pattern of overlapping shapes in green and blue. The shapes include circles, triangles, and polygons, some of which are cut out or layered to create a sense of depth and movement. The overall design is modern and abstract.

*Advanced
Mathematics*

An Incremental Development

SECOND EDITION

SAXON

Contents

<i>Preface</i>	xi
<i>Lesson 1</i> Geometry Review	1
<i>Lesson 2</i> More on Area • Cylinders and Prisms • Cones and Pyramids • Spheres	14
<i>Lesson 3</i> Pythagorean Theorem • Triangle Inequalities (1) • Similar Polygons • Similar Triangles	26
<i>Lesson 4</i> Construction	36
<i>Lesson 5</i> Exponents and Radicals • Complex Numbers • Areas of Similar Geometric Figures • Diagonals of Rectangular Solids	43
<i>Lesson 6</i> Fractional Equations • Radical Equations • Systems of Three Linear Equations	49
<i>Lesson 7</i> Inductive and Deductive Reasoning • Logic • The Contrapositive • Converse and Inverse	54
<i>Lesson 8</i> Statements of Similarity • Proportional Segments • Angle Bisectors and Side Ratios	61
<i>Lesson 9</i> Congruent Figures • Proof Outlines	67
<i>Lesson 10</i> Equation of a Line • Rational Denominators • Completing the Square	74
<i>Lesson 11</i> Circles • Properties of Circles • The Quadratic Formula	83
<i>Lesson 12</i> Angles and Diagonals in Polygons • Proof of the Chord-Tangent Theorem	90
<i>Lesson 13</i> Intersecting Secants • Intersecting Secants and Tangents • Products of Chord Segments • Products of Secant and Tangent Segments	97
<i>Lesson 14</i> Sine, Cosine, and Tangent • Angles of Elevation and Depression • Rectangular and Polar Coordinates • Coordinate Conversion	108
<i>Lesson 15</i> Assumptions • Proofs	115
<i>Lesson 16</i> Complex Fractions • Abstract Equations • Division of Polynomials	122
<i>Lesson 17</i> Proofs of the Pythagorean Theorem • Proofs of Similarity	127
<i>Lesson 18</i> Advanced Word Problems	133

<i>Lesson 19</i>	Nonlinear Systems • Factoring Exponentials • Sum and Difference of Two Cubes	138
<i>Lesson 20</i>	Two Special Triangles	145
<i>Lesson 21</i>	Evaluating Functions • Domain and Range • Types of Functions • Tests for Functions	150
<i>Lesson 22</i>	Absolute Value • Reciprocal Functions	160
<i>Lesson 23</i>	The Exponential Function • Sketching Exponentials	166
<i>Lesson 24</i>	Sums of Trigonometric Functions • Combining Functions	172
<i>Lesson 25</i>	Age Problems • Rate Problems	178
<i>Lesson 26</i>	The Logarithmic Form of the Exponential • Logarithmic Equations	183
<i>Lesson 27</i>	Related Angles • Signs of Trigonometric Functions	188
<i>Lesson 28</i>	Factorial Notation • Abstract Rate Problems	193
<i>Lesson 29</i>	The Unit Circle • Very Large and Very Small Fractions • Quadrantal Angles	197
<i>Lesson 30</i>	Addition of Vectors • Overlapping Triangles	205
<i>Lesson 31</i>	Symmetry • Reflections • Translations	211
<i>Lesson 32</i>	Inverse Functions • Four Quadrant Signs • Inverse Trigonometric Functions	219
<i>Lesson 33</i>	Quadrilaterals • Properties of Parallelograms • Types of Parallelograms • Conditions for Parallelograms • Trapezoids	233
<i>Lesson 34</i>	Summation Notation • Linear Regression • Decomposing Functions	242
<i>Lesson 35</i>	Change in Coordinates • The Name of a Number • The Distance Formula	249
<i>Lesson 36</i>	Angles Greater Than 360° • Sums of Trigonometric Functions • Boat-in-the-River Problems	256
<i>Lesson 37</i>	The Line as a Locus • The Midpoint Formula	262
<i>Lesson 38</i>	Fundamental Counting Principle and Permutations • Designated Roots • Overall Average Rate	269
<i>Lesson 39</i>	Radian Measure of Angles • Forms of Linear Equations	277
<i>Lesson 40</i>	The Argument in Mathematics • The Laws of Logarithms • Properties of Inverse Functions	286
<i>Lesson 41</i>	Reciprocal Trigonometric Functions • Permutation Notation	293
<i>Lesson 42</i>	Conic Sections • Circles • Constants in Exponential Functions	301
<i>Lesson 43</i>	Periodic Functions • Graphs of $\sin \theta$ and $\cos \theta$	308
<i>Lesson 44</i>	Abstract Rate Problems	315
<i>Lesson 45</i>	Conditional Permutations • Two-Variable Analysis Using a Graphing Calculator	319

<i>Lesson 46</i>	Complex Roots • Factoring Over the Complex Numbers	324
<i>Lesson 47</i>	Vertical Sinusoid Translations • Arctan	328
<i>Lesson 48</i>	Powers of Trigonometric Functions • Perpendicular Bisectors	333
<i>Lesson 49</i>	The Logarithmic Function • Development of the Rules for Logarithms	338
<i>Lesson 50</i>	Trigonometric Equations	342
<i>Lesson 51</i>	Common Logarithms and Natural Logarithms	346
<i>Lesson 52</i>	The Inviolable Argument • Arguments in Trigonometric Equations	349
<i>Lesson 53</i>	Review of Unit Multipliers • Angular Velocity	353
<i>Lesson 54</i>	Parabolas	358
<i>Lesson 55</i>	Circular Permutations • Distinguishable Permutations	363
<i>Lesson 56</i>	Triangular Areas • Areas of Segments • Systems of Inequalities	367
<i>Lesson 57</i>	Phase Shifts in Sinusoids • Period of a Sinusoid	375
<i>Lesson 58</i>	Distance from a Point to a Line • “Narrow” and “Wide” Parabolas	381
<i>Lesson 59</i>	Advanced Logarithm Problems • The Color of the White House	387
<i>Lesson 60</i>	Factorable Trigonometric Equations • Loss of Solutions Caused by Division	392
<i>Lesson 61</i>	Single-Variable Analysis • The Normal Distribution • Box-and-Whisker Plots	397
<i>Lesson 62</i>	Abstract Coefficients • Linear Variation	405
<i>Lesson 63</i>	Circles and Completing the Square	409
<i>Lesson 64</i>	The Complex Plane • Polar Form of a Complex Number • Sums and Products of Complex Numbers	412
<i>Lesson 65</i>	Radicals in Trigonometric Equations • Graphs of Logarithmic Functions	416
<i>Lesson 66</i>	Formulas for Systems of Equations • Phase Shifts and Period Changes	422
<i>Lesson 67</i>	Antilogarithms	426
<i>Lesson 68</i>	Locus Definition of a Parabola • Translated Parabolas • Applications • Derivation	430
<i>Lesson 69</i>	Matrices • Determinants	437
<i>Lesson 70</i>	Percentiles and z Scores	441
<i>Lesson 71</i>	The Ellipse (1)	445
<i>Lesson 72</i>	One Side Plus Two Other Parts • Law of Sines	450
<i>Lesson 73</i>	Regular Polygons	455
<i>Lesson 74</i>	Cramer’s Rule	458
<i>Lesson 75</i>	Combinations	461
<i>Lesson 76</i>	Functions of $(-t)$ • Functions of the Other Angle • Trigonometric Identities (1) • Rules of the Game	465

<i>Lesson 77</i>	Binomial Expansions (1)	472
<i>Lesson 78</i>	The Hyperbola	475
<i>Lesson 79</i>	De Moivre's Theorem • Roots of Complex Numbers	480
<i>Lesson 80</i>	Trigonometric Identities (2)	485
<i>Lesson 81</i>	Law of Cosines	489
<i>Lesson 82</i>	Taking the Logarithm of • Exponential Equations	495
<i>Lesson 83</i>	Simple Probability • Independent Events • Replacement	499
<i>Lesson 84</i>	Factorable Expressions • Sketching Sinusoids	504
<i>Lesson 85</i>	Advanced Trigonometric Equations • Clock Problems	508
<i>Lesson 86</i>	Arithmetic Progressions and Arithmetic Means	512
<i>Lesson 87</i>	Sum and Difference Identities • Tangent Identities	516
<i>Lesson 88</i>	Exponential Functions (Growth and Decay)	521
<i>Lesson 89</i>	The Ellipse (2)	526
<i>Lesson 90</i>	Double-Angle Identities • Half-Angle Identities	531
<i>Lesson 91</i>	Geometric Progressions	535
<i>Lesson 92</i>	Probability of Either • Notations for Permutations and Combinations	538
<i>Lesson 93</i>	Advanced Trigonometric Identities • Triangle Inequalities (2)	542
<i>Lesson 94</i>	Graphs of Secant and Cosecant • Graphs of Tangent and Cotangent	546
<i>Lesson 95</i>	Advanced Complex Roots	551
<i>Lesson 96</i>	More Double-Angle Identities • Triangle Area Formula • Proof of the Law of Sines • Equal Angles Imply Proportional Sides	553
<i>Lesson 97</i>	The Ambiguous Case	557
<i>Lesson 98</i>	Change of Base • Contrived Logarithm Problems	560
<i>Lesson 99</i>	Sequence Notations • Advanced Sequence Problems • The Arithmetic and Geometric Means	565
<i>Lesson 100</i>	Product Identities • More Sum and Difference Identities	570
<i>Lesson 101</i>	Zero Determinants • 3×3 Determinants • Determinant Solutions of 3×3 Systems • Independent Equations	574
<i>Lesson 102</i>	Binomial Expansions (2)	580
<i>Lesson 103</i>	Calculations with Logarithms • Power of the Hydrogen	582
<i>Lesson 104</i>	Arithmetic Series • Geometric Series	586
<i>Lesson 105</i>	Cofactors • Expansion by Cofactors	590
<i>Lesson 106</i>	Translations of Conic Sections • Equations of the Ellipse • Equations of the Hyperbola	595
<i>Lesson 107</i>	Convergent Geometric Series	600
<i>Lesson 108</i>	Matrix Addition and Multiplication	603

<i>Lesson 109</i>	Rational Numbers	610
<i>Lesson 110</i>	Graphs of arcsine and arccosine • Graphs of arcsecant and arccosecant • Graphs of arctangent and arccotangent	613
<i>Lesson 111</i>	Logarithmic Inequalities: Base Greater Than 1 • Logarithmic Inequalities: Base Less Than 1	618
<i>Lesson 112</i>	Binomial Theorem	621
<i>Lesson 113</i>	Synthetic Division • Zeros and Roots	624
<i>Lesson 114</i>	Graphs of Factored Polynomial Functions	628
<i>Lesson 115</i>	The Remainder Theorem	635
<i>Lesson 116</i>	The Region of Interest	638
<i>Lesson 117</i>	Prime and Relatively Prime Numbers • Rational Roots Theorem	643
<i>Lesson 118</i>	Roots of Polynomial Equations	647
<i>Lesson 119</i>	Descartes' Rule of Signs • Upper and Lower Bound Theorem • Irrational Roots	651
<i>Lesson 120</i>	Matrix Algebra • Finding Inverse Matrices	656
<i>Lesson 121</i>	Piecewise Functions • Greatest Integer Function	662
<i>Lesson 122</i>	Graphs of Rational Functions • Graphs that Contain Holes	665
<i>Lesson 123</i>	The General Conic Equation	671
<i>Lesson 124</i>	Point of Division Formulas	675
<i>Lesson 125</i>	Using the Graphing Calculator to Graph • Solutions of Systems of Equations Using the Graphing Calculator • Roots	679
<i>Appendix</i>	Proofs	685
	Answers	691
	Index	741

LESSON 1 *Geometry Review*

1.A

points, lines, and rays

Some fundamental mathematical terms are impossible to define exactly. We call these terms **primitive terms** or **undefined terms**. We define these terms as best we can and then use them to define other terms. The words **point**, **curve**, **line**, and **plane** are primitive terms.

A point is a location. When we put a dot on a piece of paper to mark a location, the dot is not the point because a mathematical point has no size and the dot does have size. We say that the dot is the **graph** of the mathematical point and marks the location of the point. A curve is an unbroken connection of points. Since points have no size, they cannot really be connected. Thus, we prefer to say that a curve defines the path traveled by a moving point. We can use a pencil to graph a curve. These figures are curves.



A mathematical line is a straight curve that has no ends. **Only one mathematical line can be drawn that passes through two designated points.** Since a line defines the path of a moving point that has no width, a line has no width. The pencil line that we draw marks the location of the mathematical line. When we use a pencil to draw the graph of a mathematical line, we often put arrowheads on the ends of the pencil line to emphasize that the mathematical line has no ends.



We can name a line by using a single letter (in this case, p) or by naming any two points on the line in any order. The line above can be called line AB , line BA , line AM , line MA , line BM , or line MB . Instead of writing the word *line*, a commonly used method is to write the letters for any two points on the line in any order and to use an overbar with two arrowheads to indicate that the line continues without end in both directions. All of the following notations name the line shown above. These notations are read as "line AB ," "line BA ," etc.



We remember that a part of a line is called a **line segment** or just a **segment**. A line segment contains the endpoints and all points between the endpoints. A segment can be named by naming the two endpoints in any order. The following segment can be called segment AB or segment BA .



Instead of writing the word *segment*, we can use two letters in any order and an overbar with no arrowheads to name the line segment whose endpoints are the two given points. Therefore, \overline{AB} means "segment AB " and \overline{BA} means "segment BA ." Thus, we can use either



to name a line segment whose endpoints are A and B .

The length of a line segment is designated by using letters without the overbar. Therefore, AB designates the length of segment AB and BA designates the length of segment BA . Thus, we can use either

$$AB \quad \text{or} \quad BA$$

to designate the length of the line segment shown below whose endpoints are A and B .



The words **equal to**, **greater than**, and **less than** are used only to compare numbers. Thus, when we say that the measure of one line segment is equal to the measure of another line segment, we mean that the number that describes the length of one line segment is equal to the number that describes the length of the other line segment. Mathematicians use the word **congruent** to indicate that designated geometric qualities are equal. In the case of line segments, the designated quality is understood to be the length. Thus, if the segments shown here



are of equal length, we could so state by writing that segment PQ is congruent to segment RS or by writing that the length of \overline{PQ} equals the length of \overline{RS} . We use an equals sign topped by a wavy line (\cong) to indicate congruence.

CONGRUENCE OF
LINE SEGMENTS

$$\overline{PQ} \cong \overline{RS}$$

or we could write

EQUALITY OF
LENGTHS

$$PQ = RS$$

Sometimes we will use the word *congruent* and at other times we will speak of line segments whose *measures* are equal.

A **ray** is sometimes called a **half line**. A ray is part of a line with one endpoint—the beginning point, called the **origin**—and extends indefinitely in one direction. The ray shown here begins at point T , goes through points U and X , and continues without end.



When we name a ray, we must name the origin first and then name any other point on the ray. Thus we can name the ray above by writing either "ray TU " or "ray TX ." Instead of writing the word *ray*, we can use two letters and a single-arrowhead overbar. The first letter must be the endpoint or origin, and the other letter can be any other point on the ray. Thus, we can name the ray shown above by writing either

$$\overrightarrow{TU} \quad \text{or} \quad \overrightarrow{TX}$$

These notations are read as "ray TU " and "ray TX ."

Two rays of opposite directions that lie on the same line (rays that are **collinear**) and that share a common endpoint are called **opposite rays**. Thus, rays XM and XP are opposite rays, and they are both collinear with line MX .



If two geometric figures have points in common, we say that these points are points of **intersection** of the figures. We say that the figures intersect each other at these points. If two different lines lie in the same plane and are not parallel, then they intersect in exactly one point. Here we show lines b and e that intersect at point Z .



1.B planes

A mathematical line has no width and continues without end in both directions. A mathematical plane can be thought of as a flat surface like a tabletop that has no thickness and that continues without limit in the two dimensions that define the plane. Although a plane has no edges, we often picture a plane by using a four-sided figure. The figures below are typical of how we draw planes. We label and refer to them as plane P and plane Q , respectively.



Just as two points determine a line, three noncollinear points determine a plane. As three noncollinear points also determine two intersecting straight lines, we can see that two lines that intersect at one point also determine a plane.



On the right, we see that two parallel lines also determine a plane. We say that lines that lie in the same plane are **coplanar**.

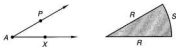
A line not in a plane is parallel to the plane if the line does not intersect the plane. If a line is not parallel to a plane, the line will intersect the plane and will do so at only one point. Here we show plane M and line k that lies in the plane. We also show line c that is parallel to the plane and line f that intersects the plane at point P .



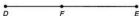
Skew lines are lines that are not in the same plane. Skew lines are never parallel, and they do not intersect. However, saying this is not necessary because if lines are parallel or intersect, they are in the same plane. Thus, lines k and f in the diagram above are skew lines because they are not both in plane M , and they do not form another plane because they are not parallel and they do not intersect.

1.C angles

There is more than one way to define an angle. An angle can be defined to be the **geometric figure** formed by two rays that have a common endpoint. This definition says that the angle is the set of points that forms the rays, and that the measure of the angle is the measure of the opening between the rays. A second definition is that the angle is the **region** bounded by two radii and the arc of a circle. In this definition, the measure of the angle is the ratio of the length of the arc to the length of the radius.



- Three times the complement of angle A is 60° less than the supplement of angle A . Find the measure of angle A .
- The ratio of students to teachers in the school was 8 to 5. If there were 1400 students, how many teachers were there?
- In a taste test, 72% of the people polled preferred cereal B. If a total of 936 people polled preferred cereal B, how many people were polled?
- Construct an angle which is congruent to $\angle ABC$, then bisect it.
- Construct a perpendicular to \overline{DE} at F .



- Construct a triangle whose sides have lengths a , b , and c .



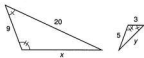
7. Solve:
$$\begin{cases} 2x + 6y = -36 \\ x - 3y = 0 \end{cases}$$

8. Solve: $6(x + x^0 - 1) = 2(-x + 8)$

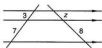
9. Add:
$$\frac{5}{x(x+1)} + \frac{4}{x+1} + \frac{3}{x}$$

10. Expand:
$$\frac{4p^3s^{-5}}{p^{-3}s} \left(2^{-1}p^{-2}s + \frac{p^2s^3}{s^{-3}} \right)$$

- The lengths of the sides of a triangle are 8 m, 6 m, and 5 m. Is the triangle a right triangle, an acute triangle, or an obtuse triangle?
- Find a , b , and c .
- Find x and y .



- Find z .



- Find x and y .



- An equilateral triangle has a perimeter of 24 cm and an area of $16\sqrt{3}$ cm². Find the altitude of the triangle.
- A sphere has a radius of 5 feet. Find the volume and surface area of the sphere.
- In the circle, O is the center. The radius of the circle is $\sqrt{8}$ meters. Find the area of the shaded sectors.
- Find the volume of the cone whose base is shown and whose altitude is 8 cm. Dimensions are in centimeters.



20. Evaluate: $x^3 - 3y^3 + 2(x - y)(x^2 + 3xy + y^2)^0$ if $x = 3$ and $y = 2$

Test 1

1. $3(90 - A) = (180 - A) - 60$

(1) $270 - 3A = 120 - A$

$$2A = 150$$

$$A = 75^\circ$$

2. $\frac{s}{t} = \frac{8}{5}$

(1) $\frac{1400}{t} = \frac{8}{5}$

$$8t = 7000$$

$$t = 875$$

3. $\frac{72}{100} = \frac{936}{T}$

(1) $72T = 93,600$

$$T = 1300$$

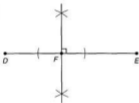
4.

(1)



5.

(1)



6.

(1)



7. $x - 3y = 0$

(1) $x = 3y$

$$2x + 6y = -36$$

$$2(3y) + 6y = -36$$

$$12y = -36$$

$$y = -3$$

$$x = 3y$$

$$x = 3(-3)$$

$$x = -9$$

8. $6(x + x^0 - 1) = 2(-x + 8)$

(1) $6x + 6 - 6 = -2x + 16$

$$8x = 16$$

$$x = 2$$

9. $\frac{5}{x(x+1)} + \frac{4}{(x+1)} + \frac{3}{x}$

(1) $= \frac{5 + 4x + 3(x+1)}{x(x+1)} = \frac{7x + 8}{x^2 + x}$

10. $\frac{4p^3s^{-5}}{p^{-3}s} \left(\frac{2^{-1}p^{-2}s}{p^3} + \frac{p^2s^3}{s^{-3}} \right)$

(1) $= \frac{4p^6}{s^6} \left(\frac{s}{2p^5} + p^2s^6 \right) = \frac{2p}{s^5} + 4p^8$

11. $c^2 \bigcirc a^2 + b^2$

(1) $8^2 \bigcirc 6^2 + 5^2$

$$64 \bigcirc 36 + 25$$

$$64 > 61$$

Since the square of the largest side is greater than the sum of the squares of the other two sides, the triangle is an **obtuse triangle**.

12.

(1)



$$c^2 = 4^2 + 3^2$$

$$c = \sqrt{16 + 9}$$

$$c = 5$$

$$\frac{4}{16} = \frac{3}{a}$$

$$12 + 4a = 48$$

$$4a = 36$$

$$a = 9$$

$$\frac{4}{16} = \frac{c}{b+c}$$

$$\frac{4}{16} = \frac{5}{b+5}$$

$$4b + 20 = 80$$

$$4b = 60$$

$$b = 15$$

11.
(15)

Statements	Reasons
1. $\overline{BT} \perp \overline{AC}$	1. Given
2. \overline{BT} bisects $\angle B$	2. Given
3. $\angle BAT \cong \angle CBT$	3. An altitude divides an angle into two congruent angles.
4. $\overline{AT} \cong \overline{CT}$	4. Reflexive axiom
5. $\triangle BAT \cong \triangle CBT$	5. SAS congruency postulate
6. $\overline{BT} \cong \overline{BT}$	6. CPCTC

12. $\frac{5}{4} = \frac{x}{8}$
(10)
 $4x = 40$
 $x = 10$

13.
(11)

$$\frac{6}{a} = \frac{7}{13}$$

$$a^2 = 78$$

$$a = \sqrt{78}$$

$$b^2 = (\sqrt{78})^2 - 6^2$$

$$b = \sqrt{78 - 36}$$

$$b = \sqrt{42}$$

$$b^2 = 7^2 + (\sqrt{42})^2$$

$$b = \sqrt{49 + 42}$$

$$b = \sqrt{91}$$

14. $\sqrt{3}\sqrt{5}\sqrt{-3}\sqrt{-5} - \sqrt{3}\sqrt{5i}\sqrt{3}\sqrt{5i^2} - \sqrt{-36}$
(10)
 $= \sqrt{3}\sqrt{3}\sqrt{5}\sqrt{5}i^2 - \sqrt{3}\sqrt{3}\sqrt{5}\sqrt{5}i^4 - 6i$
 $= (3)(5)(-1) - (3)(5)(-1)(-1) - 6i = -30 - 6i$

15. $x = \frac{130 + 150}{2} = 140$
(13)

16. $x = \frac{40 - 20}{2} = 10$
(13)

17. $3 \cdot x = 6 \cdot 2$
(11, 13)
 $3x = 12$
 $x = 4$

18. $4(4 + x) = 6(6 + 14)$
(13)
 $16 + 4x = 36 + 84$
 $4x = 104$
 $x = 26$

19. All mathematicians are engineers. The argument is valid. Bobby belongs to the set identified by the contrapositive.

20. $\frac{85 + 74 + 91 + 93 + x}{5} = 82$
(10)
 $\frac{343 + x}{5} = 82$
 $343 + x = 410$
 $x = 67$

Test 5

1. (a) $\tan 30^\circ = \frac{1}{\sqrt{3}}$
(20)

$$\frac{\sqrt{3}}{2} \tan 30^\circ = \frac{\sqrt{3}}{2} \left(\frac{1}{\sqrt{3}} \right) = \frac{1}{2}$$

(b) $\cos 60^\circ = \frac{1}{2}$

$$2\sqrt{3} \cos 60^\circ = 2\sqrt{3} \left(\frac{1}{2} \right) = \sqrt{3}$$

(c) $\sin 45^\circ = \frac{1}{\sqrt{2}}$

$$\frac{\sqrt{2}}{2} \sin 45^\circ = \frac{\sqrt{2}}{2} \left(\frac{1}{\sqrt{2}} \right) = \frac{1}{2}$$

2. Glycerol₁ + glycerol₂ = total glycerol
(10)
 $0.79(P_N) + 0.34(72) = 0.63(P_N + 72)$
 $0.79P_N + 24.48 = 0.63P_N + 45.36$
 $0.16P_N = 20.88$
 $P_N = 130.50 \text{ liters}$

Test 11

$$1. \text{Rate} = \frac{d}{m} \frac{\text{dollars}}{\text{microscope}}$$

$$\text{New rate} = \frac{d}{m} - 20 = \frac{d - 20m}{m} \frac{\text{dollars}}{\text{microscope}}$$

$$\text{New rate} \times N = \text{price}$$

$$\left(\frac{d - 20m}{m}\right)N = 2000$$

$$N = \frac{2000m}{d - 20m} \text{ microscopes}$$

$$2. 60^\circ \left(\frac{\pi}{180^\circ}\right)(120) = 40\pi = 125.66 \text{ in.}$$

$$3. y = \left(\frac{1}{4}\right)^{-2x} = \left[\left(\frac{1}{4}\right)^{-2}\right]^x = 16^x$$



$$4. {}_n P_r = \frac{n!}{(n-r)!}$$

$${}_7 P_6 - {}_3 P_2 = \frac{7!}{(7-6)!} - \frac{7!}{(7-3)!}$$

$$= \frac{7!}{1!} - \frac{7!}{4!}$$

$$= 4830$$

$$5. (a) 4 \log_2 x = 2 \log_2 16$$

$$\log_2 x^4 = \log_2 16^2$$

$$x^4 = 256$$

$$x^4 = 4^4$$

$$x = 4$$

$$(b) \log_8 216 = 3$$

$$x^3 = 216$$

$$x^3 = 6^3$$

$$x = 6$$

$$(c) \log_7 \frac{1}{49} = x$$

$$7^x = \frac{1}{49}$$

$$7^x = \frac{1}{7^2}$$

$$7^x = 7^{-2}$$

$$x = -2$$

$$6. (a) 0^\circ \leq \theta \leq 180^\circ$$

$$\text{Arccos } 1 = 0^\circ$$

$$(b) -90^\circ \leq \theta \leq 90^\circ$$

$$\text{Arcsin } (-1) = -90^\circ$$

(c)



$$-90^\circ \leq \theta \leq 90^\circ$$

$$a = \sqrt{5^2 - 3^2} = 4$$

$$\cos \left[\text{Arcsin} \left(-\frac{3}{5} \right) \right] = \frac{4}{5}$$

$$7. \log_2 3^6 + \log_4 4^5 - \log_6 6^9 = 6 + 5 - 9 = 2$$

$$8. \log_6 (x + 10) - \log_6 (x - 1) = \log_6 12$$

$$\log_6 \frac{(x + 10)}{(x - 1)} = \log_6 12$$

$$\frac{(x + 10)}{(x - 1)} = 12$$

$$x + 10 = 12x - 12$$

$$11x = 22$$

$$x = 2$$

$$9. f(x) = \frac{\sqrt{3-x}}{x^3 - 2x^2 - 8x} = \frac{\sqrt{3-x}}{x(x^2 - 2x - 8)}$$

$$= \frac{\sqrt{3-x}}{x(x-4)(x+2)}$$

$$x(x-4)(x+2) \neq 0$$

$$x \neq 0, 4, -2$$

$$3 - x \geq 0$$

$$x \leq 3$$

$$\text{Domain of } f = \{x \in \mathbb{R} \mid x \leq 3, x \neq 0, -2\}$$

$$2 \cos x + \sqrt{2} = 0$$

$$2 \cos x = -\sqrt{2}$$

$$\cos x = -\frac{\sqrt{2}}{2}$$

$$x = \frac{3\pi}{4}, \frac{5\pi}{4}$$

$$x = \frac{\pi}{3}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{4\pi}{3}$$

$$(b) \tan 2x + \frac{\sqrt{3}}{3} = 0$$

$$\tan 2x = -\frac{\sqrt{3}}{3}$$

$$2x = \frac{5\pi}{6}, \frac{11\pi}{6}, \frac{17\pi}{6}, \frac{23\pi}{6}$$

$$x = \frac{5\pi}{12}, \frac{11\pi}{12}, \frac{17\pi}{12}, \frac{23\pi}{12}$$

20.
(16)

$$H = 60 \sin 45^\circ = 42.43 \text{ m}$$

$$\text{Area} = \frac{1}{2}BH = \frac{1}{2}(25)(42.43) = 530.38 \text{ m}^2$$

Test 18

1. $y = ax^2$

(16)

$$y = \frac{1}{28}x^2$$

$$a = \frac{1}{4p}$$

$$\frac{1}{28} = \frac{1}{4p}$$

$$4p = 28$$

$$p = 7$$

The receiver should be placed 7 ft above the vertex.

2. Vertex: $(h, k) = (-3, 2)$

(16)

$$\text{Focus: } (h, k + p) = (-3, -2)$$

$$k + p = -2$$

$$(2) + p = -2$$

$$p = -4$$

$$\text{Directrix: } y = k - p = 2 - (-4) = 6$$

$$\text{Axis of symmetry: } x = h = -3$$

$$y - k = \frac{1}{4p}(x - h)^2$$

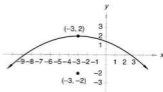
$$y - (2) = \frac{1}{4(-4)}[x - (-3)]^2$$

$$y = -\frac{1}{16}(x + 3)^2 + 2$$

$$\text{Parabola: } y = -\frac{1}{16}(x + 3)^2 + 2$$

$$\text{Directrix: } y = 6$$

$$\text{Axis of symmetry: } x = -3$$



3. (a) $\text{antilog}_2 9 = 9^2 = 81$

(17)

$$(b) \text{antilog}_8(-2) = 8^{-2} = \frac{1}{8^2} = \frac{1}{64}$$

4. $\left| \begin{matrix} x+3 & 2 \\ -3 & x \end{matrix} \right| = 10$

(18)

$$x(x + 3) - (-3)(2) = 10$$

$$x^2 + 3x + 6 = 10$$

$$x^2 + 3x - 4 = 0$$

$$(x + 4)(x - 1) = 0$$

$$x = -4, 1$$

5. $a = \frac{p\sqrt{t}}{m^2}$

(18)

$$\frac{2p\sqrt{9t}}{(6m)^2} = \frac{2p(3)\sqrt{t}}{36m^2} = \frac{6p\sqrt{t}}{36m^2} = \frac{1}{6}a$$

a is divided by 6.

6. $(7 \text{ cis } 68^\circ)(-2 \text{ cis } 82^\circ) = -14 \text{ cis } (68^\circ + 82^\circ)$

(16)

$$= -14 \text{ cis } 150^\circ = -14(\cos 150^\circ + i \sin 150^\circ)$$

$$= -14 \left[\left(-\frac{\sqrt{3}}{2} \right) + i \left(\frac{1}{2} \right) \right] = 7\sqrt{3} - 7i$$

7. $3(180 - A) = 7(90 - A) + 130$

(11)

$$540 - 3A = 630 - 7A + 130$$

$$4A = 220$$

$$A = 55^\circ$$